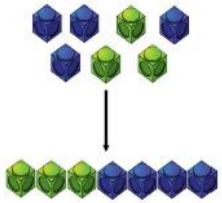
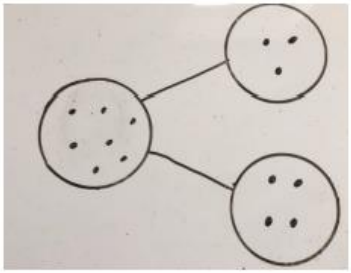
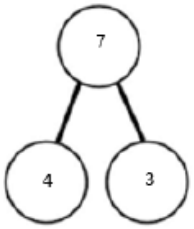
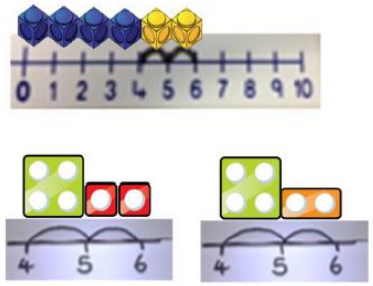
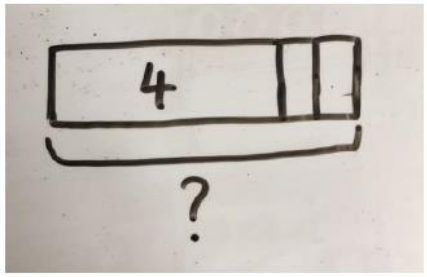
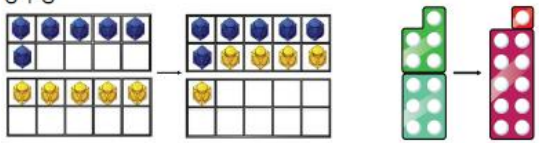
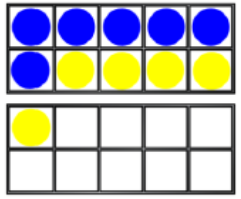
Calculation Policy: Progression in addition and subtraction (including fractions)

Addition and subtraction calculations	Early Years ELG – Add and subtract two single-digit numbers	Year 1 Add and subtract one- and two-digit numbers to 20 including zero	Year 2 Add and subtract numbers using concrete objects, pictorial representations and mentally, including • A two-digit number • A two-digit number and tens • Two two-digit numbers • Adding three one-digit numbers	Year 3 Add and subtract numbers mentally including • A three-digit number and ones • A three-digit number and tens • A three-digit number and hundreds Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	Year 4 Add and subtract numbers with up to 4 digits using formal written methods of columnar addition and subtraction where appropriate	Year 5 Add and subtract whole numbers with more than 4 digits including using formal written methods (columnar addition and subtraction Add and subtract numbers mentally with increasingly large numbers	Year 6 Perform mental calculations, including with mixed operations and large numbers. Use their knowledge of order of operations to carry out calculations involving the four operations
Addition and subtraction with fractions			Write simple fractions for example ½ of 6 = 3	Add and subtract fractions with the same denominator within one whole e.g. 1/7 + 5/7 = 6/7	Add and subtract fractions with the same denominator	Add and subtract fractions with the same denominator and denominators that are multiples of the same number	Add and subtract fractions with different denominators and mixed numbers using the concept of equivalent fractions

All images used in this calculation policy have been taken from https://whiterosemaths.com/

Addition – concrete, pictorial and abstract

Key language: sum, total, parts and wholes, plus, add altogether, more, 'is equal to' and 'is the same as'

	Concrete	Pictorial	Abstract
Combining two parts to make a whole (use a range of resources) EYFS/ Year 1	Use a range of items to combine **Combining two parts to make a whole** (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven. 
Start at the bigger number and count on using cubes. Year 1	**Counting on using number lines** using cubes or Numicon. 	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$ 
Regrouping to make 10 Year 1	**Regrouping to make 10;** using ten frames and counters/cubes or using Numicon. $6 + 5$ 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$

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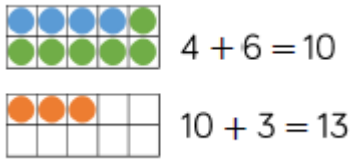
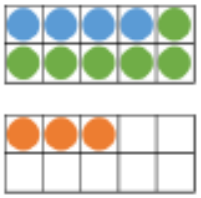
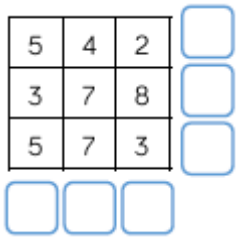
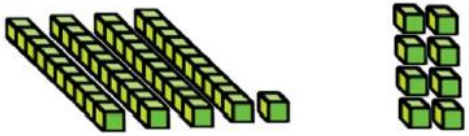
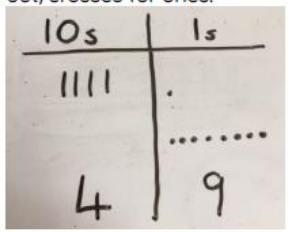
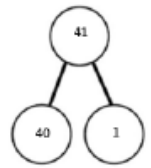
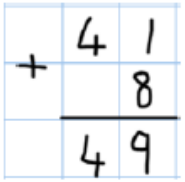
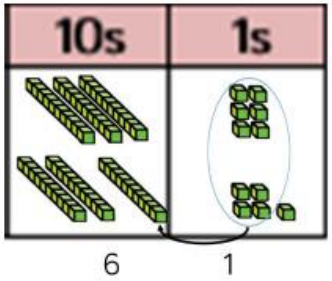
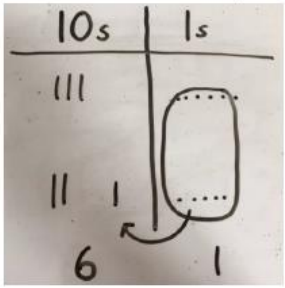
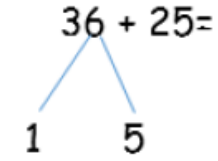
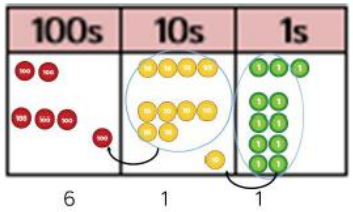
	Concrete	Pictorial	Abstract
Adding three single digits Year 2	Use ten frames and counters to add the number 4+3+6 =  	Children to draw the ten frame and counters 	Children use their knowledge of number bonds to 10 to help them add more efficiently 
Use of base 10 to combine two numbers (TO + O0 Year 2	TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.  	41 + 8  1 + 8 = 9 40 + 9 = 49 
Use of base 10 to combine two numbers (TO + TO) Year 2	TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  	Chidlren to represent the base 10 in a place value chart.  	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 36 Formal method: +25 61 1

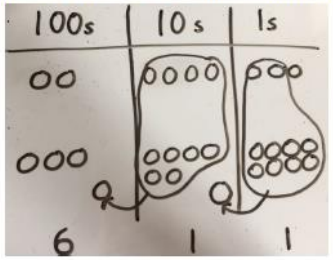
All images used in this calculation policy have been taken from https://whiterosemaths.com/

Column method and regrouping (using place value counters)
Up to 3 digits
Year 3
Up to 4 digits
Year 4
With decimals
Year 5 and Year 6

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

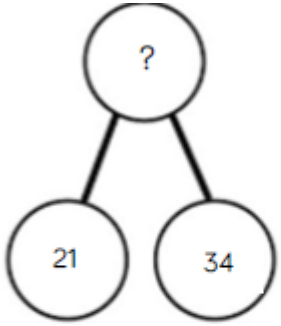


Chidren to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Conceptual variation – different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

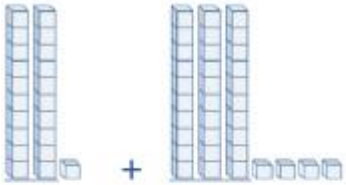
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\square$ = 21 + 34

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5

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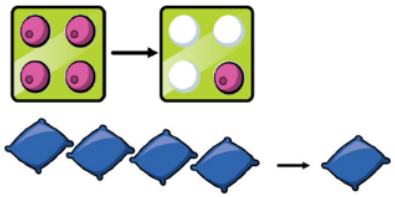
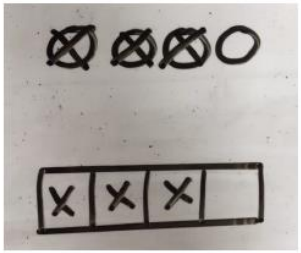
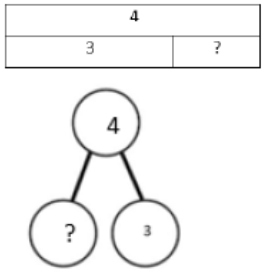
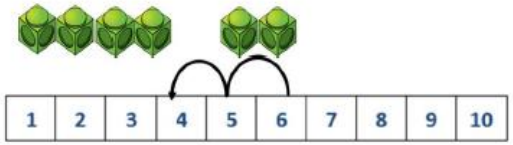
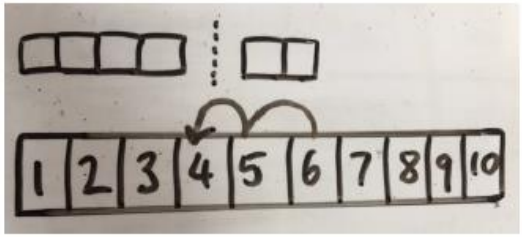
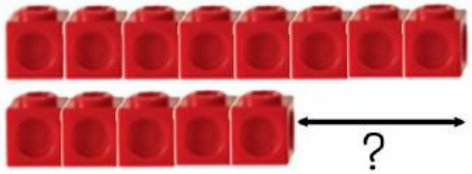
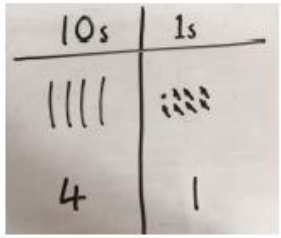
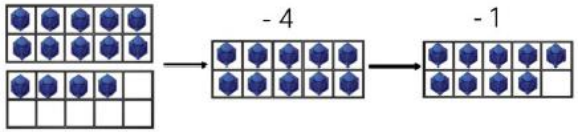
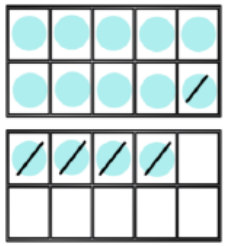
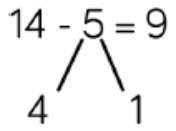
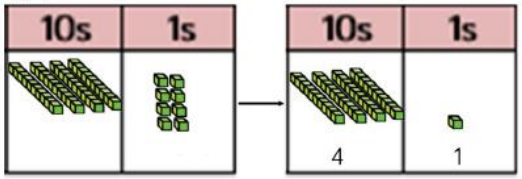
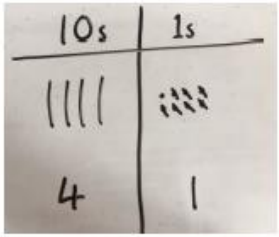
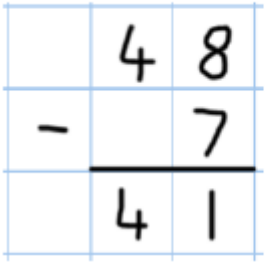
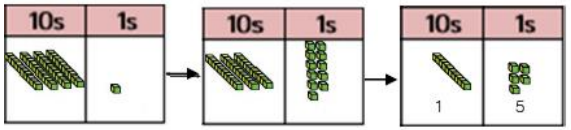
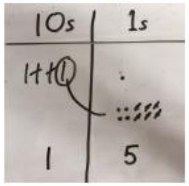
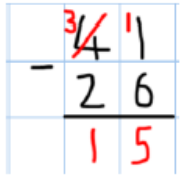
Subtraction – concrete, pictorial and abstract

Key language: take away, less than, the difference, subtract, minus, fewer and decrease

	Concrete	Pictorial	Abstract
Taking away ones EYFS/ Year 1	**Physically taking away and removing objects from a whole** (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = □ = 4 – 3 
Counting back EYFS (concrete and pictorial)/ Year 1 (concrete-abstract)	**Counting back** (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4 	Children to represent what they see pictorially e.g. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 
Finding the difference Year 1 and Year 2	**Finding the difference** (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to represent the base 10 pictorially. 	Find the difference between 8 and 5. 8 – 5, the difference is □ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

All images used in this calculation policy have been taken from https://whiterosemaths.com/

	Concrete	Pictorial	Abstract
Making 10 (e.g. subtracting 5 from 14) Year 2	Making 10 using ten frames. 14 – 5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14 – 4 = 10 10 – 1 = 9
Column method using base 10 (e.g. for TO – O) Year 3	Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 
Column method using base 10 with exchange (e.g. for 41 – 26) Year 3	Column method using base 10 and having to exchange. 41 – 26 	Represent the base 10 pictorially, remembering to show the exchange. 	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 

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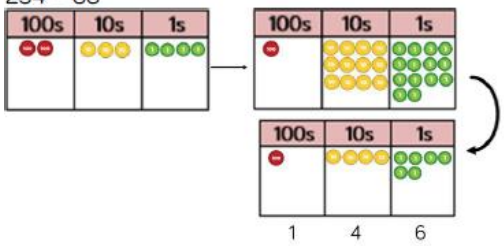
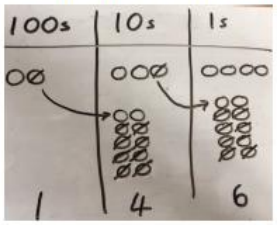
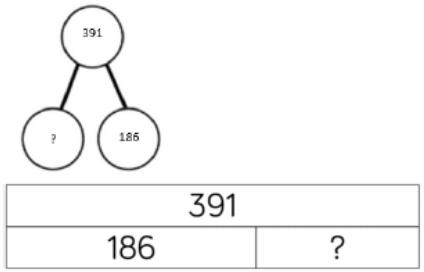
Column method and regrouping (using place value counters) Up to 3 digits Year 3 Up to 4 digits Year 4 With decimals Year 5 and 6	Column method using place value counters. 234 – 88 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal colum method. Children must understand what has happened when they have crossed out digits. $\begin{array}{r} {}^{2}\not{2}\,{}^{1}\not{3}4 \\ -\ 88 \\ \hline 6 \end{array}$

Conceptual variation – different ways to ask children to solve 391 - 186



Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$\square = 391 - 186$

$$\begin{array}{r} 391 \\ -186 \\ \hline \quad\quad \end{array}$$

What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 3\ 9\ \square \\ -\ \square\ \square\ 6 \\ \hline \square\ 0\ 5 \end{array}$$

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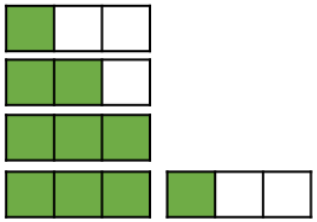
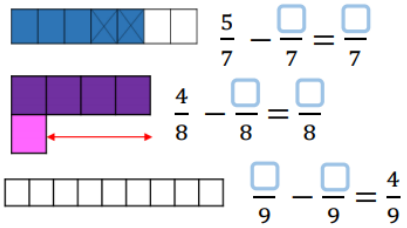
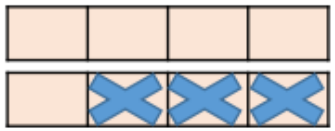
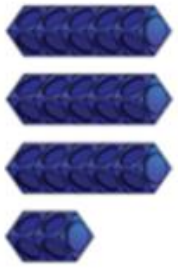
Addition and subtraction with fractions

	Concrete	Pictorial	Abstract
Counting in fractions Equivalence between quarters and half Year 2	What fraction of the cubes are yellow? 	 What would come next?	What's the same, what's different? 
Add and subtract fractions with the same denominator within one whole Year 3	Sharing chocolates in a box  $\frac{\square}{\square}+\frac{\square}{\square}=\frac{12}{12}$	 $\frac{5}{7}-\frac{\square}{7}=\frac{\square}{7}$ $\frac{4}{8}-\frac{\square}{8}=\frac{\square}{8}$ $\frac{\square}{9}-\frac{\square}{9}=\frac{4}{9}$	Find the missing fractions: $\frac{7}{7}-\frac{3}{7}=\frac{2}{7}+\frac{\square}{7}$ $\frac{\square}{9}-\frac{5}{9}=\frac{4}{9}-\frac{2}{9}$
Add and subtract fractions with the same denominator Year 4	Use strips of paper to calculate  $2-\frac{3}{4}=\frac{8}{4}-\frac{3}{4}=\frac{5}{4}=1\frac{1}{4}$	Drawing bars to calculate with fractions  $\frac{13}{5}-\frac{\square}{5}=\frac{6}{5}$	Find the difference between 2 and $\frac{2}{3}$ 2 and $\frac{2}{5}$
Add and subtract fractions with denominators that are multiples of the same number Year 5 Add mixed numbers Year 6	 Use of cubes to convert mixed numbers to improper fractions (needed for subtraction with mixed numbers in Year 6)	Use of bar model for equivalence between fractions Two children are solving $\frac{1}{3}+\frac{4}{15}$ Eva starts by drawing this model:  Alex starts by drawing this model: 	Year 5 $\frac{3}{5}+\frac{1}{10}+\frac{3}{20}$ Year 6 $34\frac{1}{9}+5\frac{2}{5}=$ Year 6 $3\frac{2}{5}-1\frac{7}{10}$

All images used in this calculation policy have been taken from https://whiterosemaths.com/

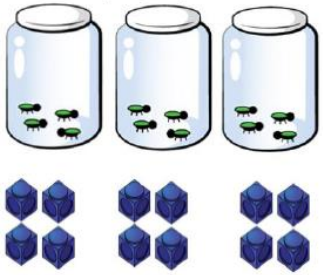
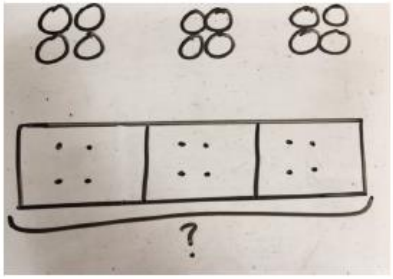
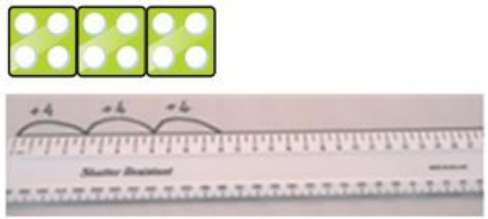
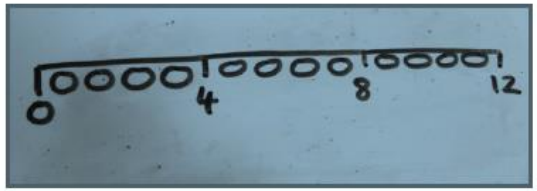
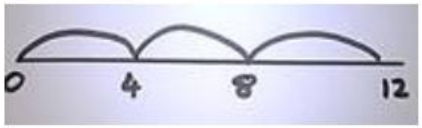
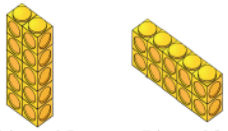
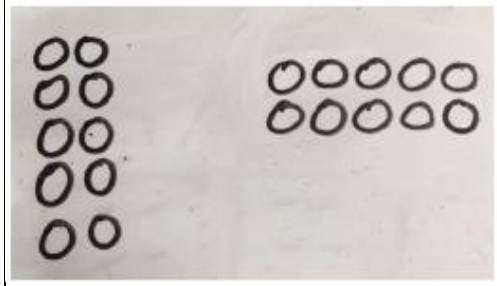
Calculation Policy: Progression in multiplication and division (including fractions)

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication and division calculations	• Count in multiples of twos, fives and tens. • Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	• Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs	• Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	• Multiply two-digit numbers and three-digit numbers by a one-digit number using formal written layout	• Multiply numbers up to 4 digits by a one or two-digit number using a formal written method, including long multiplication for two-digit numbers • Multiply and divide numbers mentally drawing upon known facts • Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	• Multiply multi-digit numbers up to 4 digits by a two-digit whole numbers using the formal written method of long multiplication • Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole numbers, fractions or rounding • divide up to 4-digit numbers by 2-digit numbers using the formal written method of short division, interpreting remainders • perform mental calculations with mixed operations and large numbers
Multiplication and division with fractions					• Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	• Multiply simple pairs of proper fractions writing the answer in its simplest form e.g. ¼ x ½ = 1/8 • Divide proper fractions by whole numbers e.g. 1/3 ÷ 2 = 1/6

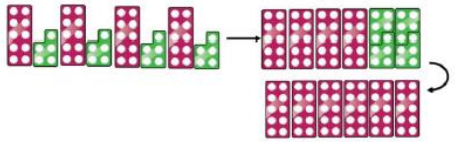
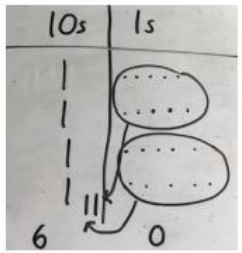
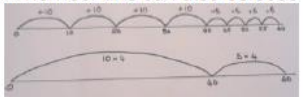
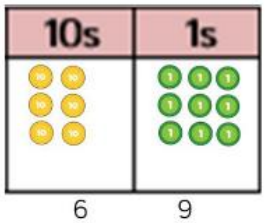
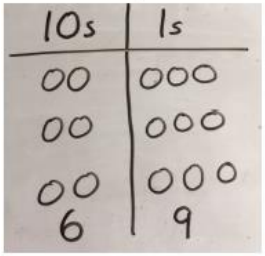
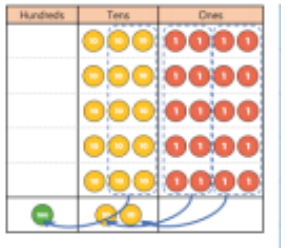
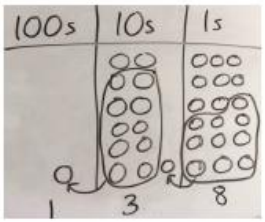
All images used in this calculation policy have been taken from https://whiterosemaths.com/

Multiplication – concrete, pictorial and abstract

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups

	Concrete	Pictorial	Abstract
Recognising, making and adding equal groups EYFS (concrete and pictorial)/ Year 1 (concrete – abstract)	Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$
Using a number line to show repeated groups EYFS/ Year 1	Number lines to show repeated groups- 3×4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. $3 \times 4 = 12$ 
Make arrays by making equal groups and building them into columns or rows Year 1 and Year 2	Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$

All images used in this calculation policy have been taken from https://whiterosemaths.com/

	Concrete	Pictorial	Abstract
Partition to multiply using numicon or base 10 Year 3/Year 4	**Partition to multiply** using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 × 15 10 5 10 × 4 = 40 5 × 4 = 20 40 + 20 = 60 A number line can also be used 
2 digit x 1 digit using place value counters, base 10 etc Year 3/Year 4	**Formal column method** with place value counters (base 10 can also be used.) 3 × 23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69
Column multiplication introduced with place value counters (2 and 3 digit x 1 digit) Year 4	34 x 5 using place value counters 	Children to represent the counters/base 10, pictorially. 	

All images used in this calculation policy have been taken from https://whiterosemaths.com/

Column multiplication (abstract only)
Year 5 and 6)

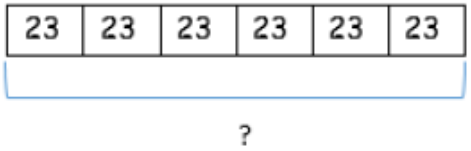
Year 5 – May need to repeat Year 4 first. 4-digit x 1 or 2 digit

		3	2	5	0
×				2	6
	1	9_{1}	5_{3}	0	0
	6	5_{1}	0	0	0
	8	4	5	0	0

Year 6 - 4 digit x 2 digit

$$\begin{array}{r} 4207 \\ \times\ 63 \\ \hline 12621 \\ {}^{2} \\ 25242{}^{1}0 \\ {}^{1}\quad{}^{4} \\ \hline \\ \hline \end{array}$$

Conceptual variation – different ways to ask children to solve 6 x 23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

□ = 6 × 23

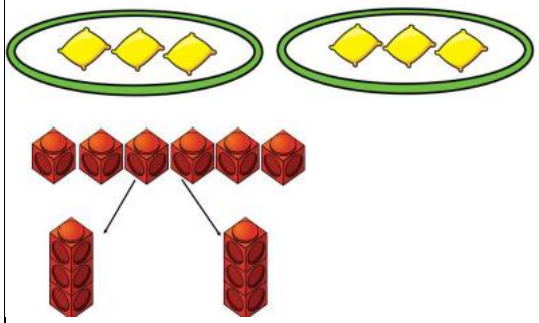
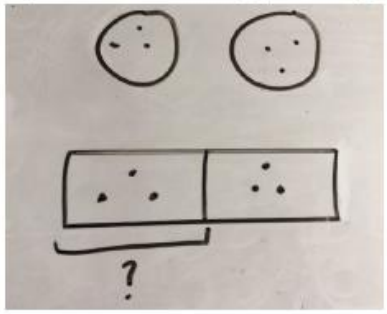
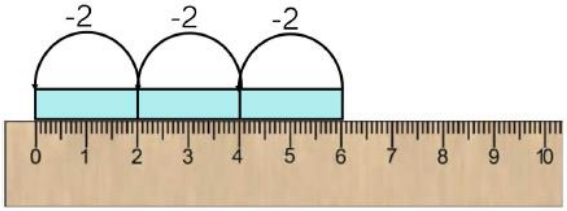
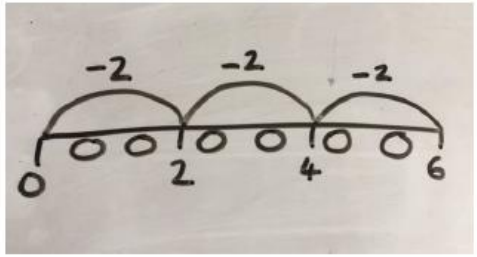
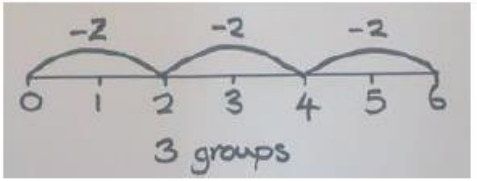
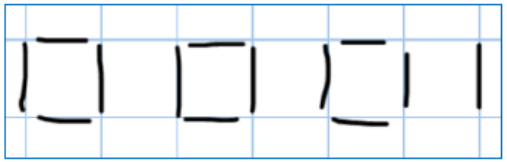
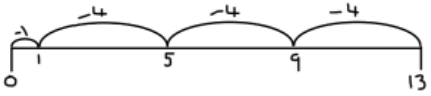
$$\begin{array}{rr} 6 & 23 \\ \times\ 23 & \times\ 6 \\ \hline & \\ \hline \end{array}$$

What is the calculation?
What is the product?

All images used in this calculation policy have been taken from https://whiterosemaths.com/

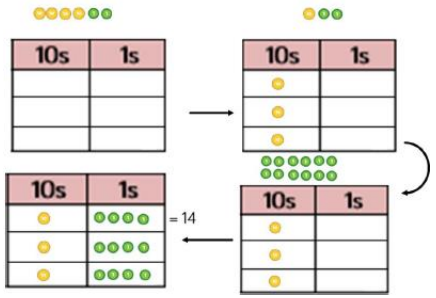
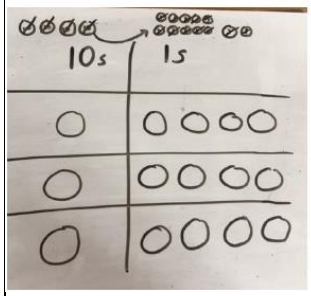
Division – concrete, pictorial and abstract

Key language: share, group, divide, divide by, half

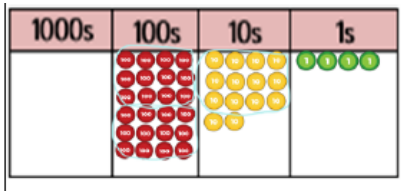
	Concrete	Pictorial	Abstract
Division as sharing into equal groups EYFS/ Year 1	**Sharing** using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Use a umber line to show subtraction of equal groups Year 1	**Repeated subtraction** using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 
Division with sharing into equal groups and finding remainders Year 1 and Year 2	**2d ÷ 1d with remainders** using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use the times table facts; they could also represer repeated addition on a number line. '3 groups of 4, with 1 left over' 

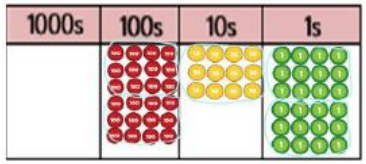
All images used in this calculation policy have been taken from https://whiterosemaths.com/

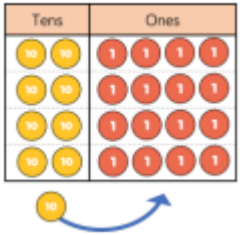
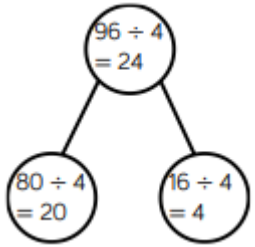
<table>
<tr><th></th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td>Dividing 2 digit number by 1 digit number

Year 2/Year 3</td><td>Sharing using place value counters.
42 ÷ 3 = 14
 
</td><td>Children to represent the place value counters pictorially.
 
</td><td>Children to be able to make sense of the place value counters and write calculations to show the process.

42 ÷ 3
42 = 30 + 12
30 ÷ 3 = 10
12 ÷ 3 = 4
10 + 4 = 14</td></tr>
<tr><td>Short division (Dividing 2 digit numbers by 1 digit numbers)

Year 3/Year 4</td><td colspan="3"> 

After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.
 

After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.
12</td></tr>
<tr><td>Short division (up to 3 digits by 1 digit)
Year 4</td><td> 
</td><td>96 ÷ 4
Using place value counters</td><td> 
</td><td>906 ÷ 3 884 ÷ 4

884 ÷ 8 489 ÷ 2

(Inclusion of remainders)</td></tr>
</table>

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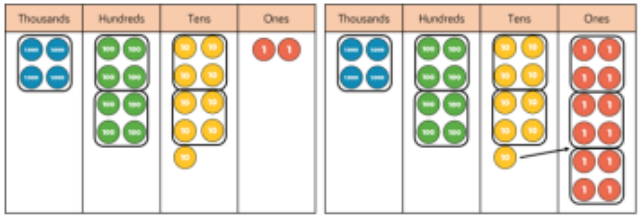
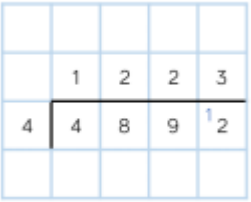
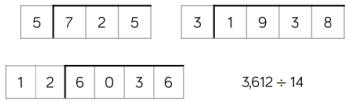
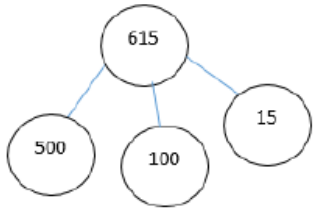
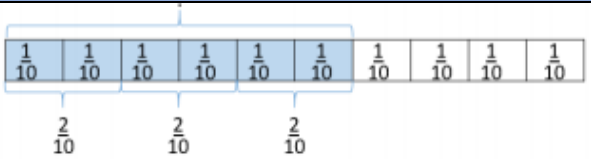
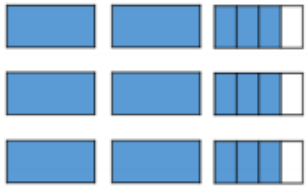
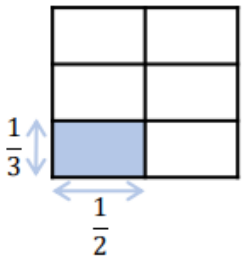
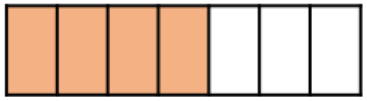
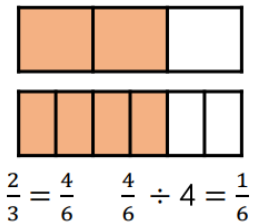
<table>
<tr>
<td>Short division (up to 4 digits by 1 digit)
Year 5</td>
<td> 
</td>
<td> 
</td>
</tr>
<tr>
<td>Long division (up to 4 digits by 2-digit number)
Year 6</td>
<td>Continue to use place value counters for modelling division
Continue to use same method for division by single digit as by a double digit – count up in multiples of the divisor to aid calculation
 
</td>
<td>512 ÷ 16
672 ÷ 21
792 ÷ 24</td>
</tr>
<tr>
<td colspan="3">Conceptual variation – different ways to ask children to solve 615 ÷ 5</td>
</tr>
<tr>
<td>Using the part whole model below, how can you divide 615 by 5 without using short division?
 
</td>
<td>I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?</td>
<td>$5\overline{)615}$
615 ÷ 5 =
☐ = 615 ÷ 5</td>
</tr>
</table>

All images used in this calculation policy have been taken from https://whiterosemaths.com/

Multiplication and division with fractions

	Pictorial	Abstract
Multiply proper fractions by whole numbers Year 5	 $3 \times \frac{2}{10}$	$\frac{2}{7} \times 2$ $4 \times \frac{3}{20}$
Multiply mixed numbers by whole numbers Year 5	 $2 \times 3 = 6$ $\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$ $6 + 2\frac{1}{4} = 8\frac{1}{4}$ $2\frac{3}{4} \times 3$	$4 \times 3\frac{3}{5}$ $2\frac{\square}{8} \times \square = 7\frac{7}{8}$
Multiply simple pairs of proper fractions Year 6	 Shade the diagram to calculate $\frac{1}{3} \times \frac{1}{2} =$	$\frac{2}{3} \times \frac{1}{4} =$ $\frac{2}{3} \times \frac{3}{4} =$ $\square \times \frac{3}{\square} = \frac{6}{12}$
Divided proper fractions by whole numbers Year 6	$\frac{3}{4} \div 3 =$  $\frac{4}{7} \div 4 =$ 	$\frac{6}{11} \div 3 =$ $\frac{9}{11} \div 3 =$ $\frac{20}{23} \div \square = \frac{5}{23}$
Divided proper fractions by whole numbers (using equivalence of fractions) Year 6	 $\frac{2}{3} = \frac{4}{6}$ $\frac{4}{6} \div 4 = \frac{1}{6}$ dividing $\frac{2}{3}$ by 4 (need to find an equivalent fraction first before dividing)	$\frac{1}{5} \div 3 =$ $\frac{2}{5} \div 3 =$ $\square \div 4 = \frac{7}{36}$

All images used in this calculation policy have been taken from https://whiterosemaths.com/

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